# A Novel Retinex Algorithm Based On Alternating Direction Optimization

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## ABSTRACT

The goal of the Retinex theory is to removed the effects of illumination from the observed images. To address this typical ill-posed inverse problem, many existing Retinex algorithms obtain an enhanced image by using different assumptions either on the illumination or on the reflectance. One significant limitation of these Retinex algorithms is that if the assumption is false, the result is unsatisfactory. In this paper, we firstly build a Retinex model which includes two variables: the illumination and the reflectance. We propose an efficient and effective algorithm based on alternating direction optimization to solve this problem where FFT (Fast Fourier Transform) is used to speed up the computation. Comparing with most existing Retinex algorithms, the proposed method solve the illumination image and reflectance image without converting images to the logarithmic domain. One of the advantages in this paper is that, unlike other traditional Retinex algorithms, our method can simultaneously estimate the illumination image and the reflectance image, the later of which is the ideal image without the illumination effect. Since our method can directly separate the illumination and the reflectance, and the two variables constrain each other mutually in the computing process, the result is robust to some degree. Another advantage is that our method has less computational cost and can be applied to real-time processing.

**Keywords:** Retinex theory, alternating direction optimization, image enhancement

## 1. INTRODUCTION

The Retinex, which synthesized by the retina and the cortex, is a theory introduced and developed mainly by Edwin Land.[1,2,3] This theory regards the human visual system as a combination of processes both in the retina and the cortex. This system can deal with illumination that changes both in brightness and color adaptively. Nowadays, the Retinex theory has been promoted in different directions, such as color correction,[4,5] contrast enhancement[6] and high dynamic range image processing.[7,8] In the field of image processing, this theory aims to eliminate the illumination effects from the observed images.

The formula of Retinex is $S = RL$, where $S$ is the observed image, $R$ is the reflectance image which values are form 0 to 1, $L$ is the illumination image. It is an ill-posed inverse problem to solve $R$ from the formula which has two variables. That means assumptions should be used to constrain the problem. There are many existing Retinex algorithms currently. Land promoted the method from a random walk to a spatially opponent operation.[9,10] These are initial approaches about solving Retinex problem. Horn proposed a Retinex algorithm of mathematical alternative in 1974.[11] He used a smoothness prior on the illumination and partial differential equation (PDE) to obtain the reflectance image. After this promotion, a common mathematical foundation, which found by Hurlbert, has some good qualities.[12] But his method cannot work for arbitrary scenes. His work can be seen as a link between Lands and Horns method. The variational framework for Retinex was first introduced by Kimmel and Elad.[13] This model controls the corresponding assumptions on both the reflectance and illumination effectively. Other kinds of this Retinex model have been proposed subsequently. Rahman used the Retinex theory and proposed the Multi-Scale Retinex with Color Restoration (MSRCR).[6] His algorithm has the effect of lightness improvement, color constancy, and scene restoration for digital image and obtained a good result.

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Most Retinex algorithms put different assumptions on either the illumination or the reflectance to solve the ill-posed problem. The result may be dissatisfactory when the scene violates the assumption. In this paper, we propose a novel Retinex algorithm using alternating direction optimization. Unlike other Retinex algorithms, we establish a simple and novel objective function based on variational framework and solve the illumination image and the reflectance image simultaneously. Since we adopt a method of alternating direction optimization to make variables mutually constraint, our algorithm has certain robustness. Moreover, we use FFT (Fast Fourier Transform) in the calculating process and this act reduce the computational cost and make our algorithm can be applied to real-time processing.

This paper is organized as follows: In section 2, we first briefly review some previous variational Retinex models, and then propose our new objective function. Section 3 is the specific solver. Experimental results are described in section 4, and the work is concluded in section 5. Section 6 is acknowledges.

## 2. THE PROPOSED VARIATIONAL RETINEX MODEL

Most Retinex algorithms take the conversion to the logarithmic domain by $s = \log S$, $l = \log L$, $r = \log R$, and thereby $s = l + r$. Kimmel and Elad firstly built the variational Retinex model by using different priors[8]:

$$\arg\min_{l} \int_{\Omega} (|\nabla l|^2 + \alpha(l-s)^2 + \beta|\nabla(l-s)|^2)dxdy \quad s.t. \;\; l \geq s,\, \langle \nabla l, \overrightarrow{n} \rangle = 0 \text{ on } \partial\Omega\,, \tag{1}$$

where $\alpha$ and $\beta$ are free positive parameters. $\Omega$ is the support of the image, $\partial\Omega$ is the boundary, and $\overrightarrow{n}$ is the normal to the boundary. This model clearly put smoothness assumption on the illumination and reflectance, meanwhile a quadratic fidelity prior between illumination and observed image. This model is a quadratic programming form and the author solved the minimization via the Euler-Lagrange equations.

Another variational Retinex model, which proposed by Dominique Zosso[14] in 2013 is:

$$\arg\min_{r} \|\nabla r - \nabla s\|_2^2 + \alpha \|r\|_2^2 + \beta \|\nabla r\|_2^2 \quad s.t. \;\; r \leq 0,\, \langle \nabla r, \overrightarrow{n} \rangle = 0 \text{ on } \partial\Omega\,, \tag{2}$$

this model is a slight rewrite of the equation (1). The goal is optimizing the reflectance image instead of the illumination. Equation (2) implies that this model is an optimization between gradient fidelity and sparsity penalties.

Inspired by the two equations mentioned above, we propose a simple and novel variational Retinex model firstly:

$$\arg\min_{R,L} \|S - RL\|_2^2 + \alpha \|DL\|_2^2 + \beta \|DR\|_2^2 + \gamma \|L - aL_0\|_2^2 \quad s.t. \;\; S \leq L\,, \tag{3}$$

where $\alpha$, $\beta$, $\gamma$ and $a$ are free positive parameters. $D$ is the difference operator at both in horizontal and vertical direction. The first penalty term ($\|S - RL\|_2^2$) constrains proximity between ($RL$) and $S$. The second penalty term ($\|DL\|_2^2$) constrains spatial smoothness on the illumination and weighted by $\alpha$. The third penalty term ($\|DR\|_2^2$) constrains spatial smoothness on the reflectance, which weighted by $\beta$ penalizes gradients.

In equation (1), $s$ is close to $l$ in logarithmic domain so the author use the penalty term ($\alpha(l-s)^2$) to $l$ constrain the value of $l$. While in image domain, $S$ may varies from $L$ greatly. For example, supposing that one pixel's value of $s$ and $l$ are 4.5 and 5 which are close, while in image domain the value of $S$ and $L$ are nearly 90 and 148. So it should take another penalty to constrain the value of $L$. We use a simple penalty term ($\|L - aL_0\|_2^2$) to constrains the illumination to avoid its intensity is too small. This term is weighted by $\gamma$ and the detail of $L_0$ will be mentioned in next section. Moreover, according to the prior, the value of $R$ is from 0 to 1, the equation should subject to: $S \leq L$.

Our proposed model is no need to the conversion to the logarithmic domain. The goal of this objective function is solve out both the illumination image and the reflectance image. We adopt a method of alternating direction optimization to solve this ill-posed problem. Since the two variables in equation (3) mutually constraint in the computing process, our algorithm has certain robustness. In section 3 we will detail the solver.

## 3. SOLVER

Equation (3) cannot be solved directly because there are two variables with only one observed data, traditional optimization methods are not usable. We adopt a special alternating direction optimization strategy to computing $R$ and $L$ iteratively. The main idea of this method is regarding one variable as known, then fixes it and updates another one. We decompose the computing process into 4 steps as following:

**Step 1: initializing $L$ and $L_0$** Since the algorithm is required one of the two variables fixed, it is natural to make a initialization. In this paper we use Gaussian low-pass filtering of the observed image as the initialization of both $L$ and $L_0$. The illumination has the property of piece-wise smoothness and the Gaussian low-pass filtering is appropriately.

**Step 2: computing $R$** When $L$ was firstly initialized and fixed, equation (3) has only one variable $R$ and is quadratic so that has a global minimum. We diagonalize derivative operators with FFT (Fast Fourier Transform) for speedup. We rewrite equation (3) as:

$$\arg\min_R \left\| R - \frac{S}{L} \right\|_2^2 + \beta \left\| DR \right\|_2^2 . \tag{4}$$

Taking the derivative of $R$ and the solution is:

$$R = \mathscr{F}^{-1}\left(\frac{\mathscr{F}(S/L)}{\mathscr{F}(1)+\beta(\mathscr{F}(D_h^T)\mathscr{F}(D_h) + \mathscr{F}(D_v^T)\mathscr{F}(D_v))}\right), \tag{5}$$

where $\mathscr{F}$ is the FFT operator, $\mathscr{F}(1)$ is the Fourier Transform of the delta function. $D_h$ and $D_v$ are the difference operation in the horizontal and vertical direction respectively. $D_h^T$ and $D_v^T$ is the transposition of $D_h$ and $D_v$. All calculations are component-wise operators. This process avoids large-matrix inversion and in the Fourier domain, the computation is faster than minimizing directly in the image space.

**Step 3: computing $L$** The computing process of $L$ is similar as $R$. After computing $R$, fixing it and rewrite equation (3) as:

$$\arg\min_L \left\| L - \frac{S}{R} \right\|_2^2 + \alpha \left\| DL \right\|_2^2 + \gamma \left\| L - aL_0 \right\|_2^2 . \tag{6}$$

Taking the derivative of $L$ and the solution is:

$$L = \mathscr{F}^{-1}\left(\frac{\mathscr{F}(\gamma a L_0 + S/R)}{\mathscr{F}(1+\gamma) + \alpha(\mathscr{F}(D_h^T)\mathscr{F}(D_h) + \mathscr{F}(D_v^T)\mathscr{F}(D_v))}\right), \tag{7}$$

all the symbols are the same meaning as in equation (5).

**Step 4: correcting $L$ and updating $L_0$** According to the prior: $S \leq L$, we simply make a correction of $L$ after is computed: $L = \max(L, S)$. Since $L_0$ is a constraint to avoid the intensity of $L$ is too small during the iterative process, we use the current result of $L$ as the $L_0$ in next computing process.

Our algorithm is sketched in **Algorithm 1**. Both gray image and color image can be enhanced by our algorithm. In next section we will show some experimental results and comparing with other methods.

## 4. EXPERIMENTAL RESULTS AND ANALYSIS

In our experiments, we process all the experimental images by Matlab R2012a on a PC with a 2.60GHz Intel Pentium Dual Core Processor. We fix $\alpha$, $\beta$, $\gamma$ and $a$ as 400, 0.1, 0.9 and 1.2 respectively in our experiments and 6-12 iterations are generally performed. Most computation is spent on FFT in equation (5) and (7). It takes about 2 seconds to process a color image with size of 750×600. Our method can be applied to real-time processing after hardware accelerating.

Fig. 1 shows our experimental results. As can be seen, our method can estimate both the illumination image and the reflectance image. It is obvious from the result that the illumination image presents the property

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**Algorithm 1**

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**Input:** observed image, parameters $\alpha$, $\beta$, $\gamma$ and $a$
**Initialization:** $L$ and $L_0 \leftarrow$ Gaussian filtering of $S$
**for iter =1:n**
fixing $L$ and $L_0$, solve $R$ in Eq. (5).
fixing $R$, solve $L$ in Eq. (7).
correcting $L \leftarrow \max(L, S)$.
update $L_0 \leftarrow L$
**end**
**Output:** illumination image $L$, reflectance image $R$

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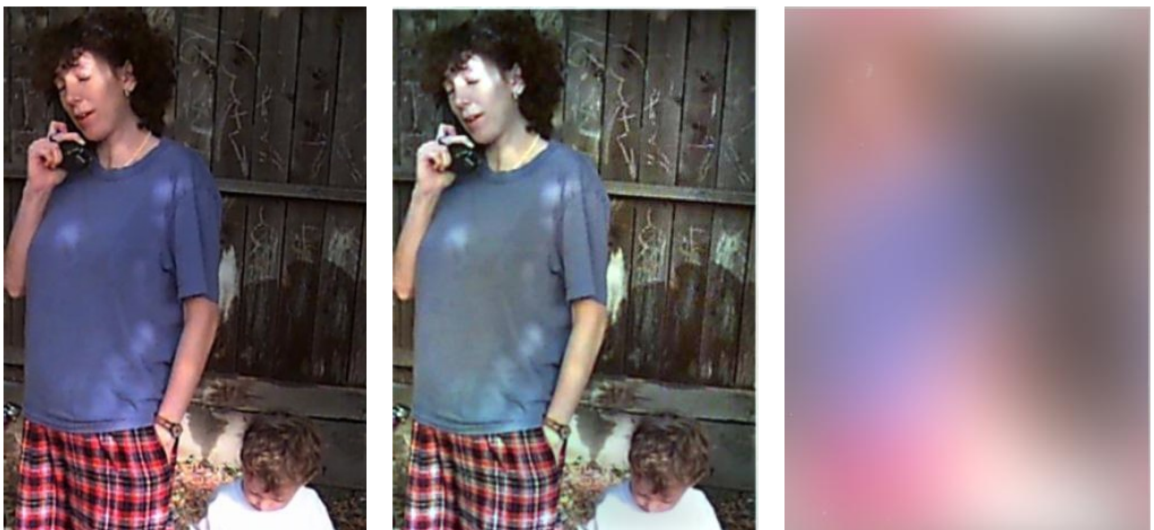

(a) (b) (c)

Figure 1. The result of color image by proposed method. (a) the observed image. (b) the reflectance image. (c) the illumination image.

of piece-wise smooth. This phenomenon accords with the illumination's priori. In the reflectance image, the woman's skin which appears red has been corrected by eliminating the affection of illumination. Moreover, the barrier in the background has been lightened and the texture can be seen clearly. This means that our algorithm has the effect of color correction and brightness enhancement.

Fig. 2 shows that our proposed method can also apply to gray image. Since the gray image does not include color information, the effect in Fig. 2 mainly appears in brightness enhancement.

Fig. 3 shows the comparison with other two classical algorithms: the grey-world assumption and MSRCR.[6] In Fig. 3(b), the grey-world assumption works nicely but still remains some blue color slightly. Meanwhile the overall brightness has no improvement due to the assumption. In Fig. 3(c), the result of MSRCR has a good performance on brightness and contrast, while the correcting of color is not ideal. In Fig. 3(d), the result of proposed method, both the color recovery and brightness enhancement are satisfying.

## 5. CONCLUSION

In this paper, we propose a novel Retinex algorithm which has good experimental results with less computational cost. We firstly build the Retinex model with the two variables, the illumination and the reflectance. Both color

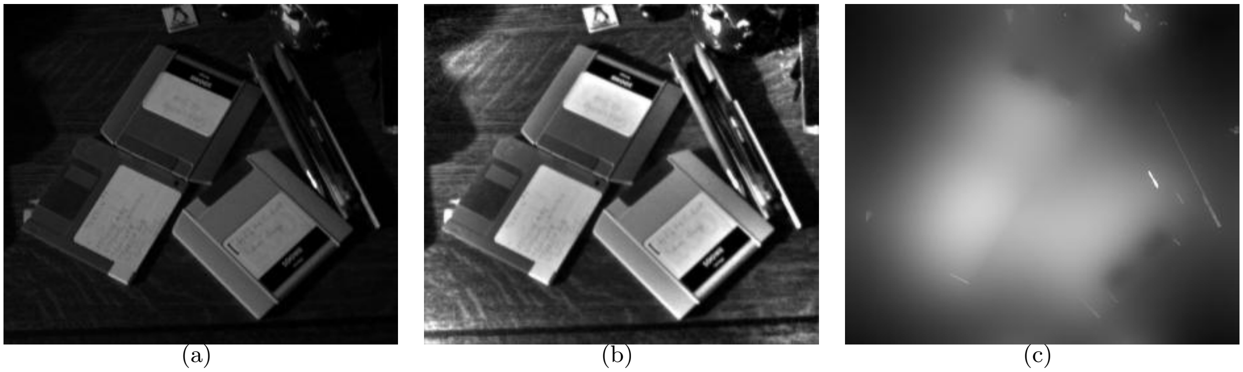


Figure 2. The result of gray image by proposed method. (a) the observed image. (b) the reflectance image. (c) the illumination image.

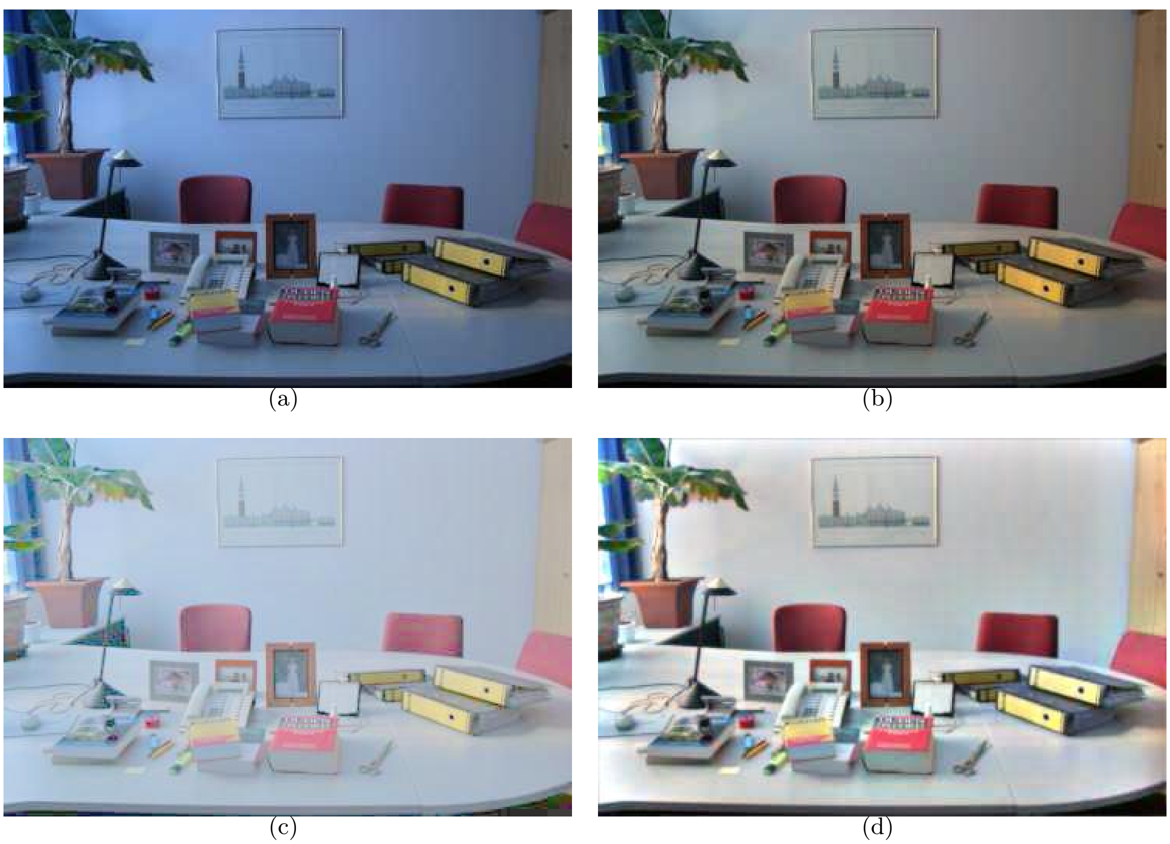


Figure 3. The comparison of different methods. (a) the observed image. (b) the grey-world assumption. (c) MSRCR.[6] (d) the reflectance image by proposed method.

images and gray images can be enhanced with our method. Since our model is all quadratic, the assumption is not perfect enough and some details will be improved. We leave these problems for further research.


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